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Intrinsic energy cut-off in diffusive shock acceleration: possible reason for non-detection of TeV-protons in SNRs

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Abstract. The linear theory of shock acceleration predicts the maximum particle energy to be limited only by the acceleration time and the size of the shock. We study the *combined* effect of acceleration nonlinearity (shock modification by accelerated particles, that *must* be present in strong astrophysical shocks) and propagation of Alfven waves that are responsible for particle confinement to the shock front. We show that wave refraction to larger wave numbers in the nonlinearly modified flow causes enhanced losses of particles in the momentum range $p_{max}/R < p < p_{max}$, where $R > 1$ is the nonlinear pre-compression of the flow and p_{max} is a conventional maximum momentum, that could be reached if there was no refraction.

1 Introduction

One of the most important parameters of the Fermi acceleration is the rate at which it operates. Indeed, what is often predicted or even observed is a power-law spectrum that cuts off due to the finite acceleration time. The cut-off momentum $p_{max}(t)$ advances with time as follows

$$\frac{dp_{max}}{dt} = \frac{p_{max}}{t_{acc}} \tag{1}$$

while the acceleration time is determined by (e.g., Axford, 1981)

$$t_{acc} = \frac{3}{u_1 - u_2} \int_{p_{min}}^{p_{max}} \left[\frac{\kappa_1(p)}{u_1} + \frac{\kappa_2(p)}{u_2} \right] \frac{dp}{p} \tag{2}$$

with u_1 and u_2 being the upstream and downstream flow speeds in the shock frame whereas κ_1 and κ_2 are the particle diffusivities in the respective media. These are the most sensitive quantities here which are determined by the rate at which particles are pitch angle scattered by the Alfven turbulence. If the latter was just background turbulence in the interstellar medium, the acceleration process would be too slow to account for the galactic cosmic rays (CRs). However it was realized (e.g., Bell, 1978) that accelerated particles must create the scattering environment by themselves generating Alfven waves via the cyclotron instability. This wave generation process proved to be very efficient (e.g., Völk *et al.*, 1984) so that the normalized wave energy density $(\delta B/B_0)^2$ is related to the partial pressure P_c of CRs that resonantly drive these waves through

$$(\delta B/B_0)^2 \sim M_A P_c/\rho u^2 \tag{3}$$

where $M_A \gg 1$ is the Alfven Mach number and ρu^2 is the shock ram pressure. Often it is assumed that the turbulence saturates at $\delta B/B_0 \sim 1$, which means that the m.f.p. of pitch angle scattered particles is of the order of their gyro-radius r_g. Then, $\kappa = \kappa_B \equiv cr_g(p)/3$, where κ_B stands for the Bohm diffusion coefficient. Hence, $t_{acc} \sim (eB/p)^{-1}(c/u_1)^2$.

However the acceleration rate (2) with $\kappa = \kappa_B$ was found to be fast enough to explain the acceleration of CRs in SNRs up to the "knee" energy $\sim 10^{15} eV$ over their life time. The analyses of Drury *et al.*, (1994) and Naito and Takahara (1994) of prospective detection of super-TeV emission from nearby SNR (they must result from the decays of π^0 mesons born in collisions of shock accelerated CR protons with the nuclei of interstellar gas) look equally optimistic. The expected fluxes were shown to be strong enough to be detected by the imaging Cherenkov telescopes. Moreover, the EGRET (Esposito *et al.*, 1996) detected a lower energy ($\lesssim \Gamma eV$) emission coinciding with some galactic SNRs. Unfortunately, despite the physical robustness of the above-mentioned predictions of emission, no statistically significant signal that could be attributed to any of the EGRET sources was detected (Buckley *et al.*, 1997). The further complication is that the region between GeV and TeV energy bands is currently uncovered by any instrument. Therefore, based on these observational results it was suggested (e.g., Buckley *et al.*, 1997) that there is probably a spectral break or even cutoff somewhere within this band.

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In this paper we attempt to understand what may happen to the spectrum provided that the acceleration is indeed fast enough to access the TeV energies over the life time of SNRs in question. Our starting point is that the fast acceleration also means that the pressure of accelerated particles becomes significant relatively early and must change the entire shock structure by this time. At the first glance this should not slow down acceleration since according to eq.(3) this increases the turbulence *level* improving thus particle confinement near the shock front and making acceleration faster. Simultaneously with that but more importantly, the upstream flow is decelerated by the pressure of CRs P_c which influences the *spectral properties* of the turbulence by affecting the propagation and excitation of the Alfven waves. This effect is twofold. First the waves are compressed in the converging plasma flow upstream and are thus blue-shifted lacking the long waves needed to keep the high energy particles diffusively bound to the accelerator. Second, and as a result of the first, at highest energies there remain relatively few particles so that the level of resonant waves is also small and hence the acceleration rate is low.

2 Basic Equations and Approximations

We use the standard diffusion-convection equation for describing the transport of high energy particles (CRs) near a CR modified shock

$$\frac{\partial f}{\partial t} + U\frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\kappa\frac{\partial f}{\partial x} = \frac{1}{3}\frac{\partial U}{\partial x}p\frac{\partial f}{\partial p} \tag{4}$$

Here x is directed along the shock normal (also the direction of the ambient magnetic field). The two quantities that control the acceleration process are the flow profile $U(x)$ and the particle diffusivity $\kappa(x,p)$. The first one is coupled to the particle distribution f through the equations of mass and momentum conservation

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}\rho U = 0 \tag{5}$$

$$\frac{\partial}{\partial t}\rho U + \frac{\partial}{\partial x}\left(\rho U^2 + P_{\rm c} + P_{\rm g}\right) = 0 \tag{6}$$

where

$$P_{\rm c}(x) = \frac{4\pi}{3}mc^2\int_{p_0}^{\infty}\frac{p^4 dp}{\sqrt{p^2+1}}f(p,x) \tag{7}$$

is the pressure of the CR gas, and $P_{\rm g}$ is the gas pressure. The lower boundary p_0 in the momentum space separates CRs from the thermal plasma and enters the equations through the magnitude of f at $p = p_0$ which specifies the injection rate. The particle momentum p is normalized to mc. We assume that the upstream region is $x > 0$ half-space, and represent the velocity profile in the shock frame as $U(x) = -u(x)$ where the (positive) flow speed $u(x)$ jumps from $u_2 \equiv u(0-)$ downstream to $u_0 \equiv u(0+) > u_2$ across the subshock and then gradually increases up to $u_1 \equiv u(+\infty) \geq u_0$ (see Fig.1a). Limiting our consideration to high Mach number shocks, $M \gg 1$, we may drop $P_{\rm g}$ term *upstream*, $x > 0$. It is retained at the subshock which, however, can be described by the conventional Rankine-Hugoniot jump condition

$$\frac{u_0}{u_2} = \frac{\gamma+1}{\gamma-1+2M_0^{-2}} \tag{8}$$

where M_0 is the Mach number in front of the subshock. For simplicity we use the adiabatic approximation, i.e., the far upstream Mach number is related to M_0 by $M_0^2 = M^2/R^{\gamma+1}$, where $R \equiv u_1/u_0$ is the flow precompression in the CR precursor.

For determining the CR diffusion coefficient κ one needs to write the wave kinetic equation for which we use the eikonal approximation

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega}{\partial k}\frac{\partial N_k}{\partial x} - \frac{\partial \omega}{\partial x}\frac{\partial N_k}{\partial k} = \gamma_k N_k \tag{9}$$

Here N_k is the number of wave quanta, ω is the wave frequency $\omega = -ku + kV_{\rm A} \simeq -ku$, k is the wave number. The left hand side has a usual Hamiltonian form that states the conservation of N_k along the lines of constant frequency $\omega(k,x) = {\rm const}$ on the k,x plane. The first term on the r.h.s. describes the wave generation on the cyclotron instability of a slightly anisotropic particle distribution. It can be expressed through its spatial gradient. The resonance condition $kp = {\rm const}$ ("resonance sharpening," e.g., Drury *et al.*, 1996 [D96]) is implied.

3 Outline of the Analysis

It is convenient to use the wave energy density normalized to $d\ln k$ and to the energy density of the background magnetic field $B_0^2/8\pi$ instead of N_k

$$I_k = \frac{k^2 V_{\rm A}}{B_0^2/8\pi}N_k \tag{10}$$

along with the partial pressure of CRs normalized to $d\ln p$ and to the shock ram pressure $\rho_1 u_1^2$

$$P = \frac{4\pi}{3}\frac{mc^2}{\rho_1 u_1^2}\frac{p^5}{\sqrt{p^2+1}}f(p,x) \tag{11}$$

Using this variables, denoting $g = P/p$, assuming a steady state and $p \gg 1$, eqs.(4,9) rewrite

$$\frac{\partial}{\partial x}\left(ug + \kappa\frac{\partial g}{\partial x}\right) = \frac{1}{3}u_x p\frac{\partial g}{\partial p} \tag{12}$$

$$u\frac{\partial I}{\partial x} + u_x p^3\frac{\partial}{\partial p}\frac{I}{p^2} = \frac{2u_1^2}{V_{\rm A}}\frac{\partial}{\partial x}P \tag{13}$$

Here the CR diffusion coefficient κ can be expressed through the wave intensity by $\kappa = \kappa_{\rm B}/I$. The difference between these equations and those used in e.g., D96 is due to the

terms with $u_x \neq 0$. Far away from the subshock where $u_x \to 0$, one simply obtains $I = 2u_1 P/V_{\rm A}$. The most important change to the acceleration process comes from the terms with $u_x \neq 0$. Indeed, let us recall first how the equation (12) may be treated in the linear case $u_x \equiv 0$ for $x > 0$. Integrating both sides between some $x > 0$ and $x = \infty$, one obtains

$$u_1 g + \frac{\kappa_0 V_{\rm A}}{2u_1}\frac{1}{g}\frac{\partial g}{\partial x} = 0 \tag{14}$$

where we denoted $\kappa_0 \equiv \kappa_{\rm B}/p \simeq const$ for $p \gg 1$. Although this equation has a formal spatial scale $l \sim \kappa_0/u_1 M_{\rm A} g$, its only solution is a power law in x

$$g \propto 1/\left(x + x_0\right) \tag{15}$$

and thus has no scale. It simply states the balance between the diffusive flux of particles escaping upstream (second term in eq.[14]) and their advection with thermal plasma in the downstream direction (the first term). As we shall see, this balance is possible not everywhere upstream and the physical reason why it appears to be so robust in the case $u_x = 0$ is that the flows of particles and waves on the x, p-plane (including the diffusive particle transport) are both directed along the x-axis. If, however, the flow modification upstream is significant ($u_x > 0$, $x > 0$), the situation changes fundamentally. Fig.1 explains how the flows of particles and waves on the x, p-plane become misaligned even though they are both advected with the thermal plasma. In fact, the flows separate from each other and, since neither of them can exist without the other (waves are generated by particles that, in turn, are trapped in the shock precursor by the waves) they both disappear in that part of the phase space where the separation is strong enough. To understand how this happens it is useful to rewrite eqs.(12-13) in the following characteristic form (we return to the particle number density f)

$$\left(u\frac{\partial}{\partial x} - \frac{1}{3}u_x p\frac{\partial}{\partial p}\right) f = -\frac{\partial}{\partial x}\kappa\frac{\partial f}{\partial x} \tag{16}$$

$$\left(u\frac{\partial}{\partial x} + u_x p\frac{\partial}{\partial p}\right)\frac{I}{p^2} = \frac{2u_1^2}{V_{\rm A}p^2}\frac{\partial}{\partial x}P \tag{17}$$

One sees from the l.h.s.'s of these equations that particles are transported towards the subshock in x and upwards in p along the family of characteristics $up^3 = const$, whereas waves move also towards the subshock but downwards in p along the characteristics $u/p = const$. As long as $u(x)$ does not significantly changes the waves and particles propagate together (along x-axis) as e.g., in the case of unmodified shock or far away from the subshock where $u_x \to 0$. When the flow compression becomes important ($u_x \neq 0$) their separation leads to decrease of both the particle and wave energy densities towards the subshock. To describe this mathematically, let us assume that the linear relation between P and I is still a reasonable approximation even if u_x is nonzero but small. (A more general case is considered in a longer version of this paper). Then, integrating eq.(12) between some $x > 0$ and $x = \infty$, instead of (14) we obtain

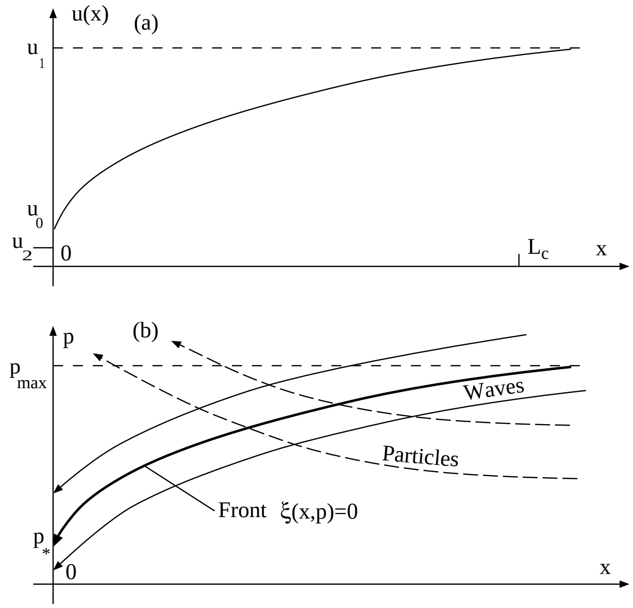

Fig. 1. (a) flow velocity; (b) characteristics of eqs.(16,17).

$$ug + u_1\frac{L}{g}\frac{\partial g}{\partial x} = -\frac{1}{3}\int_x^\infty u_x p\frac{\partial g}{\partial p}dx \tag{18}$$

In contrast to the solution of eq.(14) the length scale $L \equiv \kappa_0/2u_1 M_{\rm A}$ enters the solution of this equation. This is because it has a nonzero r.h.s. The solution of this equation changes rapidly on a scale $L \ll L_{\rm c}$ where $L_{\rm c} = \kappa(p_{\rm max})/u_1$ is the total scale height of the CR precursor. In addition to x and p, we introduce a fast (internal) variable $\xi(x, p)$ as follows

$$\xi = \frac{x - x_{\rm f}(p)}{L} \tag{19}$$

where $x = x_{\rm f}(p)$ is some special curve on the x, p plane which bounds the solution and will be specified later. We rewrite eq.(18) for $\xi\ -\ {\rm fixed},\ L \to 0$ which leads to the solution

$$g(\xi, x, p) = \frac{S(x,p)}{w(p) + e^{-S\xi/u_1}} \tag{20}$$

where $w(p) = u_{\rm f} + (1/3)p du_{\rm f}/dp$, $u_{\rm f}(p) \equiv u\left[x_{\rm f}(p)\right]$, $S(x,p) = (1/3)p du_{\rm f}/dp - (1/3)\int_x^\infty u_x p \partial G/\partial p dx$ and $G(x,p) = \lim_{\xi\to\infty} g(\xi, x, p) = S/w$. Eq.(20) describes the transition in the particle distribution between its asymptotic value $g = G$ at $\xi \to \infty$ and $g = 0$ at $\xi \to -\infty$ as a result of particle losses caused by the lack of resonant waves towards the subshock. The position of the transition front ($\xi(x, p) = 0$) is determined by the condition of non-secular behaviour of the next order asymptotic expansion and must coincide

with one of the characteristics of the operator on the l.h.s. of eq.(17), i.e.,

$$\left(u\frac{\partial}{\partial x} + u_x p \frac{\partial}{\partial p}\right)\xi(x,p) = 0$$

or

$$u_{\rm f}(p) - p\frac{du_{\rm f}}{dp} = 0$$

The reasonable choice of the concrete characteristic is based on the existence of the absolute maximum momentum $p_{\max}$ beyond which there are neither particles nor waves. That means $u_{\rm f}(p) \equiv u\left[x_{\rm f}(p)\right] = u_1 p/p_{\max}$. Likewise, the function $x = x_{\rm f}(p)$ is defined by $x_{\rm f}(p) = u^{-1}\left(u_1 p/p_{\max}\right)$.

3.0.1 External solution

While having obtained the form and the position $x = x_{\rm f}(p)$ of the narrow front in the particle distribution $g(x,p)$ we still need to calculate g to the right from the front where it decays with x. This would be the external solution $G(x,p)$ introduced above. It is clear that

$$\max_x g(x,p) \approx G(x_{\rm f}, p) \equiv G_0(p)$$

so that from eq.(18) we have the following equation

$$u_{\rm f}(p) G_0(p) = -\frac{1}{3}\int_{x_{\rm f(p)}}^{\infty} u_x p \frac{\partial G}{\partial p} dx \tag{21}$$

The most important information about $G(x,p)$ is contained in $G_0(p)$ for which from the last equation we obtain

$$\frac{\partial}{\partial p} v(p) G_0(p) + 4\frac{u_1}{p_1} G_0(p) = 0 \tag{22}$$

where we have introduced $v(p)$ by

$$v(p) = \frac{1}{G_0(p)}\int_{u_{\rm f}(p)}^{u_1} G(x,p) du(x) \tag{23}$$

Eq.(22) can be easily solved for G_0

$$G_0(p) = \frac{C}{v(p)}\exp\left(-4\frac{u_1}{p_{\max}}\int\frac{dp}{v(p)}\right) \tag{24}$$

(where C is a constant). However, the function v depends on the solution itself. Nevertheless, it can be calculated prior to determining G_0 and therefore, this solution may be written in a closed form. In the case $p \simeq p_{\max}$one obtains (the shape of the cut-off)

$$G_0 \sim \left(p_{\max} - p\right)^3 \tag{25}$$

In the rest of the x,p-domain where $x_{\rm f}(p) < x < L_{\rm c}$ and p is not close to $p_{\max}$, we may assume that the CR diffusion coefficient is close to its Bohm value since in contrast to the phase space region $x \approx x_{\rm f}(p)$ at each given x, p there are waves generated along the entire characteristic of eq.(13) passing through this point of the phase space and occupying an extended region of the CR precursor, Fig.1. We may adopt then the asymptotic high Mach number solution (Malkov and Drury 2001 [MD] and references therein)

$$G(x,p) \simeq G_0(p)\exp\left(-\frac{1}{\kappa_{\rm B}}\int_{x_{\rm f}}^{x} u dx\right)$$

Using the linear approximation for $u(x)$ [MD] $u = u_0 + u_1 x/L'_{\rm c}$ with $L'_{\rm c} = \kappa_{\rm B}(\hat{p})/u_1$, where it is implied that the maximum contribution to the particle pressure comes from some momentum $p \sim \hat{p}$, we can express v in the form of an error function integral

$$v(p) \simeq \int_{u_{\rm f}}^{u_1} du \exp\left[-\frac{L'_{\rm c}}{2u_1\kappa_{\rm B}(p)}\left(u^2 - u_{\rm f}^2\right)\right]$$

The further algebra simplifies in two limiting cases leading to the following asymptotic expressions for the particle distribution

$$G_0(p) = \begin{cases} \frac{C}{\sqrt{p}}\exp\left(-8\sqrt{\frac{2p}{\pi p_{\max}}}\right), & \frac{p_{\max}}{p} \gg 1 \\ \left(p_{\max} - p\right)^3 & p \lesssim p_{\max} \end{cases}$$

This result is valid for $p \geq p_* \equiv p_{\max}/R = p_{\max} u_0/u_1 \lesssim \hat{p}$, whereas for $p < p_*$ one still has $G_0 = C/p^{q(p)-3}$, where the detailed calculations of $q(p)$, C and R may be found in MD.

4 Conclusions

Refraction to shorter wave lengths in a nonlinearly modified flow results in a spectral break at $p = p_* = p_{max}/R$ where $R = u_1/u_0 \gg 1$ is the nonlinear pre-compression of the flow. The spectrum rollover ($p_{max}/R < p < p_{max}$) is described by $f \propto p^{-q} e^{-\sqrt{p/\bar{p}}}$with even faster decay at $p \sim p_{\max}$ while an approximate power-law $p^{-q(p)}$ is still valid at $p < p_*$. Since R itself is proportional to p_*, the spectral break p_* should grow slower than $\sqrt{p_{max}}$. Due to the lack of waves at p_{max}, it advances slower than in the Bohm case.

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